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DYNAMIC FEATURES OF A SPHERICAL GAS BUBBLE IN FLUID

V.K. Kedrinskiy

ABSTRACT: An analysis is made of a spherical gas bubble in both compressible and incompressible fluids. The movements of the bubble wall are analyzed in relation to pressure waves and conditions imposed by an incompressible fluid. The spherical symmetry of pulsations in a compressible nonviscous fluid is examined.

Most of the problems related to the dynamics of a fluid which contains gas bubbles are essentially concerned with the character of the pulsation of individual bubbles. An additional pressure field, determined by these pulsations, has a decided effect on the general state of the bubble medium in many cases. We will be concerned below with certain features of the pulsation of a spherical gas bubble in compressible and incompressible fluids. /120*

(1) The movements of spherical bubble walls in an incompressible liquid, without considering the viscosity, is determined by the following question:

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = (P(R) - P(t))/\rho \tag{1.1}$$

Here $P(R)$ is the pressure inside the bubble, $P(t)$ is the applied pressure, ρ is the density of the fluid, R is the radius of the bubble; the dot signifies the total time derivative. When $P(t)$ = const and the bubble is contracted adiabatically, we can easily obtain the following from (1.1):

$$(R_0/R_*)^{3\gamma-3} = 1 + A(\gamma - 1) \quad (A = P/P_0) \tag{1.2}$$

(where γ is the adiabatic characteristic, R_* is the minimum radius for the density, R_0 is the original radius, P_0 is the original pressure in the bubble). We can also determine the time for density contraction in the same way:

$$t = 0.915 R_0 \sqrt{\rho/P} \tag{1.3}$$

But we usually should be concerned with the pressure, which

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*Numbers in the margin indicate pagination in the foreign text.

depends substantially on the time. In this case, we cannot determine either the time or the degree of contraction of the bubble directly from (1.1).

The results of a numerical solution to (1.1) in dimensionless form are given in [1]:

$$y\frac{d^2y}{dz^2}+\frac{3}{2}\left(\frac{dy}{dz}\right)^2=\mu\left(\frac{1}{y^{3\gamma}}-Ae^{-z}\right)\quad\left(y=\frac{R}{R_0},\ z=\frac{t}{\tau},\ \mu=\left(\frac{\tau}{R_0\sqrt{\rho/p_0}}\right)^2\right)$$

Here μ is the dimensionless parameter which determines the relationship between the time constant for pressure decrease and the characteristic time for bubble contraction by a constant pressure P_0. The calculations were made for the case of waves with an exponential profile of A = 10, 100, and 1000, during a change in μ from 0.01 to 1000. The analysis showed that the pulsation of a bubble affected by a pressure with various τ complies with a definite rule /12

$$(t_2^*/\tau_2)=(1.2-k\sqrt{\mu_2/\mu_1})^k(\mu_1/\mu_2)^{1/2}(t_1^*/\tau_1) \tag{1.4}$$

Here t_2* is the unknown time for bubble contraction for waves with τ_2 in an unknown t_1* for a wave of the same amplitude but with τ_1 (the pressure at the front of the wave results in the following equation for t_1*). The index k is determined from the condition,

$$10^k=\left(\frac{\mu_1}{\mu_2}\right)^{1/2} \tag{1.5}$$

As we can see from (1.4), the relationship among the times for bubble contraction in pressure waves with various τ with an accuracy up to a constant coefficient is determined by the square root of the relationship for the parameters μ which are characteristic for these waves. Equation (1.4) gives us the possibility (for example, knowing the time for bubble contraction by a wave with constant pressure at the front) to determine the time for compression of this bubble by a wave with the same amplitude but with very small τ, and μ_1 is chosen so that it satisfies (1.3).

Another characteristic for the pulsating bubble is the minimum contraction radius. The relationships between the minimum radius, the amplitude of the pressure, and the characteristics of a constant pressure wave can, by analogy with (1.2), take the following form:

$$\left(\frac{R_0}{R_*}\right)^{3\gamma-3}=1+\frac{\mu A^2(\gamma-1)}{1+\mu A} \tag{1.6}$$

We can see from this that, for $\mu \to \infty$, i.e. for waves with a constant pressure at the front, this expression is the same as in (1.2).

The calculations for various A and μ(R_0 = 1 cm) according to (1.4) and (1.6) are given in Tables 1 and 2, where they are compared to the data in [1] (the results of machine calculation are given in Table 1, and the results of calculations according to (1.4) and (1.6) are given in Table 2). The relationships obtained in (1.4) and (1.6) are very useful for approximate estimations of the principal characteristics of the bubble pulsation in an incompressible fluid affected by a pressure which changes greatly with time.

TABLE 1

A	μ	γ	t_0 sec	$(R^*/R_0)_1$	$(R^*/R_0)_2$
10	∞ (10)	1.4	0.00316	0.262	0.262
10	1	1.4	0.001	0.307	0.280
10	0.1	1.4	0.000316	0.404	0.400
10	0.01	1.4	0.0001	0.738	0.772
100	∞ (10)	1.4	0.00316	0.048	0.046
100	1	1.4	0.001	0.048	0.0457
100	0.1	1.4	0.000316	0.053	0.0490
100	0.01	1.4	0.0001	0.074	0.0793
10	∞ (10)	1.33	0.00316	0.247	0.235
10	1	1.33	0.001	0.277	0.250
10	0.1	1.33	0.000316	—	0.377
10	0.01	1.33	0.0001	0.732	0.768
100	∞ (10)	1.33	0.00316	0.030	0.0294
100	1	1.33	0.001	0.031	0.0306
100	0.1	1.33	0.000316	0.035	0.0323
100	0.01	1.33	0.0001	0.052	0.0570
1000	∞ (1)	1.33	0.001	0.00304	0.00302
1000	0.1	1.33	0.000316	0.00317	0.00302
1000	0.01	1.33	0.0001	0.00358	0.00333
10	∞ (10)	1.67	0.00316	0.382	0.361
10	1	1.67	0.001	0.413	0.376
10	0.1	1.67	0.000316	0.498	0.48
10	0.01	1.67	0.0001	0.764	0.78
100	∞ (10)	1.67	0.00316	0.123	0.121
100	1	1.67	0.001	0.126	0.122
100	0.1	1.67	0.000316	0.136	0.127
100	0.01	1.67	0.0001	0.1[illegible]	0.170
1000	∞ (1)	1.67	0.001	0.0[illegible]	0.0387
1000	0.1	1.67	0.000316	0.040	0.0390

(2) A limit to the problems on bubble pulsation in rafts of /12
an incompressible fluid results in significant divergences from the actual characteristics of pulsation to the calculated ones if we examine the cases in which the walls of the cavities reach velocities on the order of the sound velocity. The latter takes place, for example, in the problems of cavitation and the phenomena accompanying it. The theory of collapses in hollow bubbles during its very first formulation led us to conclude that it is necessary to consider the contractability, in view of the large values for velocities and pressures obtained as a result of the burst. The contraction of gas-filled cavities affected by great pressures is also observed in the same way. Let us examine the spherically symmetric problem of a gas bubble pulsation in a compressible viscous

fluid. We will replace the velocity of the particle $U^{(1)}$ by the slope of the velocity potential ϕ, and we will write the equations thus:

$$\frac{\partial}{\partial t}(-\nabla\varphi)+(\mathbf{U}\cdot\nabla)\mathbf{U}=-\frac{\nabla P}{\rho}, \quad \nabla\cdot\mathbf{U}=-\frac{1}{\rho}\frac{d\rho}{dt} \tag{2.1}$$

Here P is the pressure, ρ is the density of the fluid. Integration of (2.1) gives

$$-\frac{\partial\varphi}{\partial t}+\frac{r^2}{2}=-\int_{P_\infty}^{P}\frac{dP}{\rho}=-h \tag{2.2}$$

if h is the difference in enthalpies between point r and infinity. We will assume that P_∞ at infinity is constant, that the velocity and the potential of velocity at infinity disappear, and that ρ is a function only of pressure. Using

$$\varphi=\frac{1}{r}f\left(t-\frac{r}{c}\right) \tag{2.3}$$

the equation can be written in this way

TABLE 2

A	μ	γ	τ_0 sec	t_1^*/τ_1	t_2^*/τ_0
10	1000	1.4	0.0316	0.01	0.01
10	100	1.4	0.0100	0.0325	0.0323
10	10	1.4	0.00316	0.105	0.110
10	1	1.4	0.001	0.35	0.37
10	0.1	1.4	0.000316	1.27	1.38
10	0.01	1.4	0.0001	5.11	5.00
100	10	1.4	0.00316	0.0294	0.0294
100	1	1.4	0.001	0.0939	0.0950
100	0.1	1.4	0.000316	0.308	0.323
100	0.01	1.4	0.0001	1.098	1.130
10	100	1.33	0.01	0.0325	0.0323
10	10	1.33	0.00316	0.105	0.110
10	1	1.33	0.001	0.350	0.373
10	0.01	1.33	0.0001	5.11	5.00
100	10	1.33	0.00316	0.0294	0.0294
100	1	1.33	0.001	0.0939	0.0950
100	0.1	1.33	0.000316	0.307	0.323
100	0.01	1.33	0.0001	1.094	1.130
1000	1	1.33	0.001	0.029	0.029
1000	0.1	1.33	0.000316	0.093	0.094
1000	0.01	1.33	0.0001	0.304	0.319
10	10	1.67	0.00316	0.105	0.110
10	1	1.67	0.001	0.35	0.373
10	0.1	1.67	0.000316	1.19	1.380
10	0.01	1.67	0.0001	5.11	5.00
100	10	1.67	0.00316	0.0295	0.0295
100	1	1.67	0.001	0.0944	0.095
100	0.1	1.67	0.000316	0.309	0.323
100	0.01	1.67	0.0001	1.100	1.130
1000	1	1.67	0.001	0.029	0.029
1000	0.1	1.67	0.000316	0.093	0.094

$$r\left(h + \tfrac{1}{2}U^2\right) = f'(t - r/C) \tag{2.4}$$

Equations (2.4) and (2.5) show that $r\phi$ and $r(h + \frac{1}{2}U^2)$ in an acoustic approximation expand with the velocity C (local sound velocity). On the basis of the fact that the velocities of the fluid can reach values on the order of the sound velocity, Kirkwood [2] made an assumption as to the propagation $r(h + \frac{1}{2}U^2)$ at a velocity $C + U$. On the basis of these assumptions we will write [3, 4]: /123

$$\frac{\partial}{\partial t}\left[r\left(h + \frac{U^2}{2}\right)\right] = -(C + U)\frac{\partial}{\partial r}\left[r\left(h + \frac{U^2}{2}\right)\right] \tag{2.5}$$

Opening (2.5) by using

$$\frac{dU}{dt} = -\frac{\partial h}{\partial r}, \quad \frac{\partial U}{\partial r} + \frac{2U}{r} = -\frac{1}{C^2}\frac{dh}{dt} \quad \left(\frac{d}{dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial r}\right) \tag{2.6}$$

we will obtain the rule for movement of the gas bubble wall

$$RR''\left(1 - \frac{R'}{C}\right) + \frac{3}{2}R'^2\left(1 - \frac{1}{3}\frac{R'}{C}\right) = H\left(1 + \frac{R'}{C}\right) + \frac{RH'}{C}\left(1 - \frac{R'}{C}\right) \tag{2.7}$$

However, we must explain to what degree (2.7) agrees with the precise equations for the flow (2.1). It is obvious that for a more complete determination of the possibilities in Kirkwood's assumption, we must examine the case of contraction for a hollow cavity, which entails examining the behavior of the function obtained in a large range of velocities for the cavity walls (from 0 to ∞).

A numerical integration of (2.1), made by Kanter [5] for a spherically symmetrical hollow cavity in water, showed great velocities for flow near the collapsing point. It was found that the cavity radius in this case is proportional to $(-t)^n$ ($t = 0$ is the moment of collapse). The flow in the areas surrounding the collapsing point is described by a self-similar solution from which the value for n is determined. The value for n was found equal to 0.5552 in [5, 6]. Writing (2.7) for the case of a hollow cavity (i.e. assuming that C = const and H = const), we obtain

$$RR''\left(1 - \frac{R'}{C}\right) + \frac{3}{2}R'^2\left(1 - \frac{1}{3}\frac{R'}{C}\right) = H\left(1 + \frac{R'}{C}\right) \tag{2.8}$$

$$\left(\frac{R_0}{R}\right)^3 = \left(1 - \frac{1}{3}\frac{R'}{C}\right)^4\left[1 + \frac{3}{2}\frac{C^2}{-H}\left(\frac{R'}{C}\right)^2\right] \tag{2.9}$$

Substituting Kanter's solution [5] in the form of $R \sim At^n$ into (2.9), we can easily obtain the value of n for $t \to 0$. It is equal to 0.666. And for the case of a nonviscous fluid, $n = 0.4$, i.e. the behavior of the cavity wall in the areas surrounding the collapsing point is as far from Kanter's results as in the nonviscous case. Repetition of the arguments mentioned above for the acoustic case results in

$$\boxed{R\ddot{R}\left(1+\frac{2\dot{R}}{C}\right)+\frac{3}{2}\dot{R}^{2}\left(1-\frac{4}{3}\frac{\dot{R}}{C}\right)=H+\frac{R\dot{H}}{C}\left(1-\frac{\dot{R}}{C}+\frac{\dot{R}^{2}}{C^{2}}\right)} \tag{2.10}$$

which in the case of a hollow cavity gives the value $n = 0.5$, i.e. an accurate solution lies between the acoustic and Kirkwood's calculation. We should note that (2.10) is none other than Hering's equation [2], although the latter was obtained by a method other than the one described above. From the values obtained for n we can conclude that the velocity of expansion for the value of $r(h + U^2/2)$ lies between C and $C + U$. We will assume that the expansion occurs with a velocity $C + \alpha U$, where α = const. In this case we have

$$\boxed{\left(\frac{R_0}{R}\right)^3=\left[1+\frac{3}{2}\frac{C^2}{-H}\left(\frac{\dot{R}}{C}\right)^2\right]\left[1+\left(\alpha-\frac{4}{3}\right)\frac{\dot{R}}{C}\right]^{-\frac{4}{3}/\left(\alpha-\frac{4}{3}\right)}} \tag{2.11}$$

Substituting $R = At^{9.555}$ into this, we can easily obtain the value for α in the areas surrounding the collapsing point (at an infinite wall velocity). It is equal to 0.57. An analysis of the behavior of (2.11) for various $\dot{R}/C$ showed that, in a certain approximation (we found α, corresponding to Kanter's curve, for each moment), α will be a monotonically decreasing function of $\dot{R}/C$, which changes its value from 1 to 0.57 when $R \to 0$. However, this does not exclude the possibility for an approximate description of the collapsing process by using a certain constant value for α. We must note that (2.11) is fairly adequate, since for various α, it can be converted into Kirkwood's equation ($\alpha = 1$), an equation with acoustic approximation ($\alpha = 0$), or any intermediate term.

TABLE 3 /124

$-\dot{R}/C$	$(R/R_0)_1$	$(R/R_0)_2$	$(R/R_0)_3$	$(R/R_0)_4$	$(R/R_0)_5$
1.46	$1.48 \cdot 10^{-2}$	$1.41 \cdot 10^{-2}$	$1.68 \cdot 10^{-2}$	$1.54 \cdot 10^{-2}$	$2.40 \cdot 10^{-2}$
2.05	$1.00 \cdot 10^{-2}$	$9.56 \cdot 10^{-3}$	$1.24 \cdot 10^{-2}$	$1.10 \cdot 10^{-2}$	$1.92 \cdot 10^{-2}$
2.50	$7.84 \cdot 10^{-3}$	$7.48 \cdot 10^{-3}$	$1.03 \cdot 10^{-2}$	$8.92 \cdot 10^{-3}$	$1.68 \cdot 10^{-2}$
2.93	$6.42 \cdot 10^{-3}$	$6.08 \cdot 10^{-3}$	$8.90 \cdot 10^{-3}$	$7.55 \cdot 10^{-3}$	$1.51 \cdot 10^{-2}$
3.56	$5.00 \cdot 10^{-3}$	$4.66 \cdot 10^{-3}$	$7.40 \cdot 10^{-3}$	$6.10 \cdot 10^{-3}$	$1.33 \cdot 10^{-2}$
4.00	$4.27 \cdot 10^{-3}$	$3.97 \cdot 10^{-3}$	$6.62 \cdot 10^{-3}$	$5.37 \cdot 10^{-3}$	$1.23 \cdot 10^{-2}$
4.62	$3.56 \cdot 10^{-3}$	$3.22 \cdot 10^{-3}$	$5.80 \cdot 10^{-3}$	$4.50 \cdot 10^{-3}$	$1.12 \cdot 10^{-2}$
5.50	$2.85 \cdot 10^{-3}$	$2.49 \cdot 10^{-3}$	$4.90 \cdot 10^{-3}$	$3.73 \cdot 10^{-3}$	$1.00 \cdot 10^{-2}$
6.88	$2.14 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$	$3.94 \cdot 10^{-3}$	$2.87 \cdot 10^{-3}$	$8.55 \cdot 10^{-3}$
9.50	$1.43 \cdot 10^{-3}$	$1.03 \cdot 10^{-3}$	$2.96 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$6.90 \cdot 10^{-3}$
11.30	$1.14 \cdot 10^{-3}$	$7.66 \cdot 10^{-4}$	$2.44 \cdot 10^{-3}$	$1.59 \cdot 10^{-3}$	$6.13 \cdot 10^{-3}$
19.50	$5.70 \cdot 10^{-4}$	$2.91 \cdot 10^{-4}$	$1.43 \cdot 10^{-3}$	$8.20 \cdot 10^{-4}$	$4.26 \cdot 10^{-3}$
50.00	$1.72 \cdot 10^{-4}$	$4.90 \cdot 10^{-5}$	$5.62 \cdot 10^{-4}$	$2.56 \cdot 10^{-4}$	$2.28 \cdot 10^{-3}$
80.00	$9.42 \cdot 10^{-5}$	$2.10 \cdot 10^{-5}$	$3.51 \cdot 10^{-4}$	$1.41 \cdot 10^{-4}$	$1.67 \cdot 10^{-3}$
100.00	$7.16 \cdot 10^{-5}$	$1.34 \cdot 10^{-5}$	$2.81 \cdot 10^{-4}$	$1.07 \cdot 10^{-4}$	$1.44 \cdot 10^{-3}$

TABLE 3. (con't)

$-R^{\cdot}/C$	$(R/R_0)_1$	$(R/R_0)_2$	$(R/R_0)_3$	$(R/R_0)_4$	$(R/R_0)_5$
200.00	$2.97\ 10^{-5}$	$3.33\ 10^{-6}$	$1.41\ 10^{-4}$	$4.38\ 10^{-5}$	$9.08\ 10^{-4}$
300.00	$1.78\ 10^{-5}$	$1.49\ 10^{-6}$	$9.40\ 10^{-5}$	$2.62\ 10^{-5}$	$6.92\ 10^{-4}$
400.00	$1.24\ 10^{-5}$	$8.40\ 10^{-7}$	$7.02\ 10^{-5}$	$1.82\ 10^{-5}$	$5.70\ 10^{-4}$
500.00	$9.27\ 10^{-6}$	$5.33\ 10^{-7}$	$5.60\ 10^{-5}$	$1.36\ 10^{-5}$	$4.90\ 10^{-4}$
600.00	$7.37\ 10^{-6}$	$3.72\ 10^{-7}$	$4.70\ 10^{-5}$	$1.07\ 10^{-5}$	$4.35\ 10^{-4}$
700.00	$6.06\ 10^{-6}$	$2.72\ 10^{-7}$	$4.01\ 10^{-5}$	$8.93\ 10^{-6}$	$3.94\ 10^{-4}$
800.00	$5.11\ 10^{-6}$	$2.09\ 10^{-7}$	$3.52\ 10^{-5}$	$7.50\ 10^{-6}$	$3.58\ 10^{-4}$
900.00	$4.38\ 10^{-6}$	$1.65\ 10^{-7}$	$3.13\ 10^{-5}$	$6.47\ 10^{-6}$	$3.33\ 10^{-4}$
1000.0	$3.85\ 10^{-6}$	$1.34\ 10^{-7}$	$2.82\ 10^{-5}$	$5.65\ 10^{-6}$	$3.10\ 10^{-4}$
10000.0	$2.07\ 10^{-7}$	$1.34\ 10^{-9}$	$2.82\ 10^{-6}$	$3.00\ 10^{-7}$	$6.60\ 10^{-5}$
100000.0	$1.11\ 10^{-8}$	$1.34\ 10^{-11}$	$2.82\ 10^{-7}$	$1.62\ 10^{-8}$	$1.44\ 10^{-5}$

The following data are given in Table 3:

(1) Numerical integration of the flow- and Kanter- equations;
(2) Calculation with Kirkwood's assumption according to the equation:

$$\left(\frac{R_0}{R}\right)^3=\left[1+\frac{3}{2}\frac{C^2}{-H}\left(\frac{R^{\cdot}}{C}\right)^2\right]\left[1-\frac{1}{3}\frac{R^{\cdot}}{C}\right]^4 \quad (2.12)$$

(3) Calculation of the acoustic variation

$$\left(\frac{R_0}{R}\right)^3=\left[1+\frac{3}{2}\frac{C^2}{-H}\left(\frac{R^{\cdot}}{C}\right)^2\right]\left[1-\frac{4}{3}\frac{R^{\cdot}}{C}\right] \quad (2.13)$$

(4) Calculation for assuming an expansion with velocity $C + 0.6U$

$$\left(\frac{R_0}{R}\right)^3=\left[1+\frac{3}{2}\frac{C^2}{-H}\left(\frac{R^{\cdot}}{C}\right)^2\right]\left[1-0.73\frac{R^{\cdot}}{C}\right]^{1.82} \quad (2.14)$$

(5) Calculation for a nonviscous fluid

$$\left(\frac{R_0}{R}\right)^3=\left[1+\frac{3}{2}\frac{C^2}{-H}\left(\frac{R^{\cdot}}{C}\right)^2\right] \quad (2.15)$$

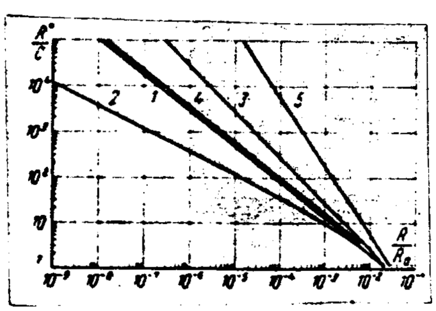

Fig. 1.

The dependence of the value for $R^{\cdot}/C$ on R/R_0, plotted according to the data of the table is shown in Figure 1. As we could easily note, Kirkwood's assumption agrees with the actual values for $R^{\cdot}/C$ on the order of 1 and less [7], and simply does not agree for greater velocities. This is very natural if we consider that the velocity $C + U$ is valid for the plane case. The acoustic variation also differs rather substantially from Kanter's curve, and in the range for $R^{\bullet}/C$ from 1 to 10 it differs even more from precise data than Kirkwood's curve. An introduction of the conditional coefficient α into the velocity of the expansion allows us to make certain estimations on the behavior of the value for $C + U$ as the velocity of the bubble wall increases.

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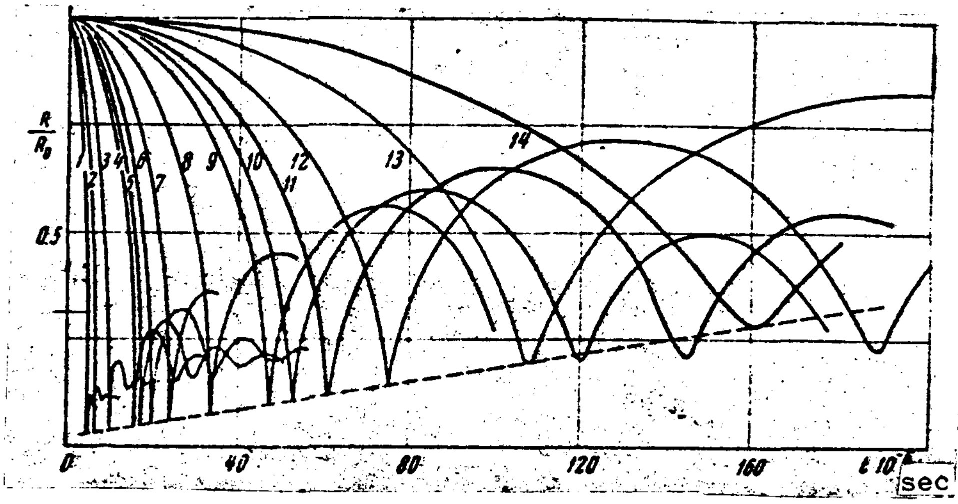

Fig. 2.

Taking the result obtained into account, we calculated the pulsation of a spherical bubble with a diamter of 1 cm affected by a suddenly-induced constant pressure whose amplitude changed from 10 to 18000 atm. An air bubble was examined for an original pressure of 1 atm. The calculation was made on a computer according to (2.7). The equation for the state of the water was considered according to [2]. The calculation results are given in Figure 2. The numbers correspond to amplitude of pressures in atm - 18000, 9000, 3000, 1000, 800, 600, 400, 200, 100, 80, 60, 40, 20, 10. The dotted line connects all the first pulsation minima. We can easily find from the graph that R^*/R_0 is directly proportional to the time for contraction of the cavity

$$\boxed{R^* / R_0 = At_{t_*} + 0.025} \quad (2.16)$$

The contraction time t^* is rather accurately determined by (1.3), and $A = 5/3\ 10^3\ \text{sec}^{-1}$ is easily found from the graph.

Certain experiments were conducted in a hydrodynamic shock tube by determining the pulsations of air bubbles at pressures of several hundred atmospheres with a weakly-changing pressure on the surface. The method for conducting the experiment and a description of the arrangement are given in [8], and the characteristic time-rebound of a bubble pulsation is also given there. The data on the degree and time of contraction correspond to the calculated values.

In conclusion, the author would like to express his gratitude to L. Trokhan for his substantial aid in the computer calculations.

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